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FQHE and Jain's approach on the torus

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Abstract

By using the explicit knowledge of the lowest energy single particle wave functions in the presence of an *arbitrary* magnetic field, we extend to the case of a torus Jain's idea of looking at the FQHE as a manifestation of an integer effect for *composite fermions*. We show that this can be realized thanks to a redefinition of the vacuum state that is explicitly collective in nature. We also discuss the relationship of this approach with the hierarchical scheme and with the characterization of the Hall states in terms of $W_{1+\infty}$ algebras and 2D conformal field theories.

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Since the fractional quantized Hall effect (FQHE)¹ was first observed by Tsui, Stormer and Gossard in 1982² considerable experimental progress has been made, performing measurements with samples of higher mobility under stronger magnetic fields and at lower temperature.

The experimental results relative to the sequence of filling factors $\nu = p/(2p \pm 1)$ and $\nu = p/(4p \pm 1)$ seem to give strong support to the idea first suggested by Jain³ of looking at the FQHE for electrons as a manifestation of the integer effect (IQHE) for *composite fermions*, obtained by attaching to each electron an even number of flux units opposite to the external magnetic field. In fact, not only the most prominent Hall plateaux are seen at the fillings of the principal sequence $p/(2p \pm 1)$, but also the energy gaps measured for this sequence correspond to the cyclotron energies relative to the reduced magnetic field $B - B_{1/2}$ ⁴. Furthermore, Halperin, Lee and Read⁵, following a point of view closely related to Jain's approach, have stressed the rôle of the state at $\nu = 1/2$, arguing for the existence of many features of a Fermi surface and computing an anomaly in surface acoustic wave propagation in agreement with the results of recent experiments⁶.

The emerging picture for the FQHE seems to be quite simpler and clearer than the traditional hierarchical scheme⁷, which assigns the Hall plateaux of the principal series to different levels of the hierarchy. Still two essential aspects of the FQHE, namely the presence of quasi-particle vortex excitations, carrying fractional charge and statistics⁸, and the topological nature of the Hall conductance⁹, do not appear in a natural way in Jain's approach. On the other hand it is well known that both properties, at least for the "Laughlin states" corresponding to fillings $\nu = 1/m$, are encoded in the m -fold degeneracy of the electron wave function on the torus¹⁰. In fact the Hall conductance can be simply related to the behaviour of the zeros of the wave function under a change of boundary conditions and, due to the m -fold degeneracy, one needs a change of phase between 0 and $2m\pi$ along one of the two cycles of the torus to achieve a full winding. Furthermore, by using 2D conformal field theory techniques, one can relate the m -fold degeneracy of the wave function to the existence of m distinct $g = 1$ *vacua*, each corresponding to a possible excitation with a charge multiple of the elementary anyonic charge $1/m$ ¹¹.

It is then a natural and relevant problem to analyze how the correct multi-electron wave functions on the torus can be obtained within Jain's approach, in view of the change of degeneracy going from integer to fractional filling. In this paper we discuss this problem by using the explicit knowledge of the lowest energy single particle wave functions on the torus in the presence of an *arbitrary* magnetic field. As a consequence of this simple and direct approach one can also understand, already for the case of the plane, why it is possible to look at the FQHE, that is a multi-particle effect, as at an IQHE, usually thought of as a single particle phenomena, for the composite fermions. This can be realized thanks to a redefinition of the vacuum state that is explicitly collective in nature.

In the following we will be mainly concerned with the case $\nu = 1/m = 1/(2k + 1)$, corresponding to the filling of the first Landau level for the composite fermions. We will briefly discuss the general case at the end of the paper.

Let us start by recalling that for an arbitrary magnetic field orthogonal to the (x, y) plane in the gauge $\nabla \cdot \vec{A} = 0$ one can define $\varphi(x, y)$ such that:

$$\frac{e}{c\hbar}A_y = \partial_x\varphi \quad ; \quad \frac{e}{c\hbar}A_x = -\partial_y\varphi \quad (1)$$

and

$$\frac{e}{c\hbar}B(x, y) = (\partial_x^2 + \partial_y^2)\varphi \quad (2)$$

The Hamiltonian describing the motion of an electron in the plane is given by the Pauli operator:

$$H = -\frac{\hbar^2}{2m}\{(\partial_x + i\partial_y\varphi)^2 + (\partial_y - i\partial_x\varphi)^2 + \sigma_3(\partial_x^2 + \partial_y^2)\varphi\} \quad (3)$$

For an electron with spin aligned with the magnetic field the states

$$\psi(x, y) = e^{-\varphi}\chi(z) \quad (4)$$

where $\chi(z)$ is an arbitrary analytic function of $z = x + iy$, have all zero energy. These are actually the lowest energy states as the Hamiltonian is non negative:

$$H = \frac{\hbar^2}{m}a^\dagger a \quad (5)$$

where

$$a^\dagger = \sqrt{2}(-\partial + \partial\varphi) \quad ; \quad a = \sqrt{2}(\bar{\partial} + \bar{\partial}\varphi) \quad (6)$$

and, as usual, $\partial = \partial/\partial z$, $\bar{\partial} = \partial/\partial \bar{z}$.

If the magnetic field is uniform it is convenient to work in magnetic units $\lambda^2 = c\hbar/(eB)$. Then a and a^\dagger are ordinary annihilation and creation operators: $[a, a^\dagger] = 1$. For example in the symmetric gauge the potential is given by $\varphi_S = \frac{z\bar{z}}{4}$ and

$$a^\dagger = \sqrt{2}(-\partial + \frac{\bar{z}}{4}) \quad a = \sqrt{2}(\bar{\partial} + \frac{z}{4}) \quad (7)$$

It is also convenient to define the operators

$$b^\dagger = \sqrt{2}(-\bar{\partial} + \bar{\partial}\varphi_S) = \sqrt{2}(-\bar{\partial} + \frac{z}{4}) \quad (8)$$

$$b = \sqrt{2}(\partial + \partial\varphi_S) = \sqrt{2}(\partial + \frac{\bar{z}}{4}) \quad (9)$$

which act again as a couple of annihilation and creation operators, $[b, b^\dagger] = 1$, commuting with a , with a^\dagger and with the Hamiltonian. A complete set of states is then given by

$$\psi_{l,n} = \frac{(a^\dagger)^l (b^\dagger)^n}{\sqrt{l!}\sqrt{n!}} e^{-\frac{z\bar{z}}{4}} \quad (10)$$

When dealing with the first Landau level one can consider b and b^\dagger as acting on the analytic part of the wave function, leading, with suitable normalization, to the Fock-Bargmann (FB) representation:

$$b = \partial \quad b^\dagger = z \quad (11)$$

Then, if $b\chi_0 = 0$, a basis for the analytic part of the wave functions can be written as:

$$\chi_n = \frac{(b^\dagger)^n}{\sqrt{n!}} \chi_0 = \frac{z^n}{\sqrt{n!}} \quad (12)$$

For the Laughlin wave function at filling $\nu = 1$, describing a uniform charge distribution in the thermodynamical limit, one has:

$$\chi_{\nu=1}(z_1, z_2, \dots, z_{N_e}) = \prod_{i < j} (z_i - z_j) = \sum_P (-1)^P (b_1^\dagger)^{n_{i_1}} (b_2^\dagger)^{n_{i_2}} \dots (b_{N_e}^\dagger)^{n_{i_{N_e}}} \chi_0 \quad (13)$$

Here $n_{i_1}, n_{i_2} \dots n_{i_{N_e}}$ is a permutation, P , of the non negative integers smaller than the electron number N_e .

Let us now study the motion of an electron in the presence of the uniform magnetic field B and of an infinitely thin magnetic flux tube located at z' of strength $-2k$ in units $\phi_0 = hc/e$. The lowest energy states will still be of the form given by eq. 4 where the potential φ can be taken as

$$\varphi = \varphi_S + h(z) = \frac{z\bar{z}}{4} - 2k \ln(z - z') \quad (14)$$

The non-analytic part of the wave function is unchanged with respect to the uniform case. Then, we can define b' and b'^\dagger along the same line of eqs. 8 and 9, namely:

$$b'^\dagger = \sqrt{2}(-\bar{\partial} + \bar{\partial}\varphi) = \sqrt{2}(-\bar{\partial} + \bar{\partial}\varphi_S) \quad (15)$$

$$b' = \sqrt{2}(\partial + \partial\varphi) = \sqrt{2}[\partial + \partial(\varphi_S + h)] \quad (16)$$

They commute with a and a^\dagger up to delta-function terms and their action on the analytic part is given by

$$b' = \partial + \partial h(z) = \partial - \frac{2k}{z - z'} \quad , \quad b'^\dagger = z \quad (17)$$

The analytic part of the new states can then be written as

$$\chi'_n = \frac{(b'^\dagger)^n}{\sqrt{n!}} \chi'_0 \quad (18)$$

where the new vacuum state χ'_0 is defined by $b'\chi'_0 = 0$, i.e. $\chi'_0 = (z - z')^{2k}$.

The presence of a zero of order $2k$ at z' is expected, due to the delta function singularity of the magnetic field. As a consequence, in Jain's composite fermions approach, no zero is wasted¹² as flux tubes are attached to the particles and their presence plays the same rôle of a singular repulsive two body potential. If at filling $\nu = 1/m = 1/(2k + 1)$ the electrons are taken to be uniformly distributed with

density n_e , the composite fermions see an effective magnetic field $B_{eff} = B - 2k\phi_0 n_e$ and have an effective filling $\nu_{eff} = 1$. More specifically, generalizing eq. 17, we define

$$b_i = \partial_i - 2k\partial_i \sum_{j \neq i} \ln(z_i - z_j) = \partial_i - 2k \sum_{j \neq i} \frac{1}{z_i - z_j} \quad , \quad b_i^\dagger = z_i \quad (19)$$

Due to eq. 13, 18 and 19, the analytic part of the wave function will be given by

$$\chi_{\nu=1/m}(z_1, z_2, \dots, z_{N_e}) = \sum_P (-1)^P (b_1^\dagger)^{n_{i_1}} (b_2^\dagger)^{n_{i_2}} \dots (b_{N_e}^\dagger)^{n_{i_{N_e}}} \chi_c = \prod_{i < j} (z_i - z_j)^m \quad (20)$$

where $b_i \chi_c = 0$ or

$$\chi_c = \prod_{i < j} (z_i - z_j)^{2k} \quad (21)$$

Eq. 20 gives the Laughlin wave function at filling $\nu = 1/m$ and the choice of the *collective vacuum state* χ_c turns out to be consistent with the requirement of uniform distribution.

Notice that for such a filling we could have directly bound m units of flux to the electrons turning them into *composite bosons* moving in an average zero field. We would have then defined the b_i and b_i^\dagger operators, as in eq. 19, but with $2k$ turned into $m = 2k + 1$. The composite bosons would condense in a collective vacuum state defined now by:

$$\left(\partial_i - m \sum_{j \neq i} \frac{1}{z_i - z_j} \right) \chi(z_1, z_2, \dots, z_{N_e}) = 0 \quad (22)$$

Eq. 22, that leads again to the Laughlin wave function, can be seen as the linear differential equation for the correlators of abelian Wess-Zumino fields¹³ of conformal weight $m/2$, making contact with the analysis of the Hall effect in terms of 2D conformal field theory¹⁴.

We close our discussion of Jain's approach on the plane by recalling that the Laughlin state at $\nu = 1$ can be characterized as a highest weight vector of the $W_{1+\infty}$ algebra of the area preserving non singular diffeomorphisms¹⁵ generated by

$$\mathcal{L}_{m,n} = \sum_{i=1}^{N_e} (b_i^\dagger)^{m+1} (b_i)^{n+1} \quad , \quad n, m \geq -1 \quad (23)$$

where the b_i 's and b_i^\dagger 's are in the FB representation, eq. 11, that is

$$\mathcal{L}_{m,n} \chi_{\nu=1}(z_1, z_2, \dots, z_{N_e}) = 0 \quad , \quad n > m \geq -1 \quad (24)$$

In the $\nu = 1/m$ case the operators b_i 's and b_i^\dagger 's, given by eq. 19, satisfy creation and annihilation commutation relations up to delta terms in the electrons relative coordinates:

$$[b_i, b_i^\dagger] = 1 + 2k\pi \sum_{j \neq i} \delta(z_i - z_j) \quad , \quad [b_i, b_j^\dagger] = -2k\pi \delta(z_i - z_j) \quad j \neq i \quad (25)$$

However, acting on the collective vacuum they are algebraically equivalent to those given by the FB representation 11, as the contribution of the delta terms in eq. 25 will vanish. Taking into account the structure of the Laughlin states, eqs. 13 and 20, we conclude that an algebraic characterization of incompressibility, analogous to eq. 24, still holds for filling $\nu = 1/m$ ^{15,16}, i.e.:

$$\mathcal{L}_{m,n}\chi_{\nu=1/m}(z_1, z_2, \dots, z_{N_e}) = 0 \quad n > m \geq -1 \quad (26)$$

where the $\mathcal{L}_{m,n}$ are still given by eq. 23 but in terms of the new b 's.

2. In order to derive the Laughlin wave function on the torus¹⁰ in the framework of Jain's approach let us start by recalling the structure of the doubly periodic single particle wave functions in an uniform magnetic field.

To make the argument as simple as possible we take a square torus of length L in magnetic units, and, to have non trivial solutions, we assume the total flux through the surface of the torus to be an integer measured in quantum flux units, i.e. $L^2 = 2\pi N_s$. In the gauge $\vec{A} = y\hat{x}$ the eigenfunctions of the first Landau level may be written in the form

$$\psi(x, y) = e^{-\frac{1}{2}y^2} f(z) \quad (27)$$

As the vector potential has a discontinuity along the line $y = L$, $\psi(x, y)$ is periodic under translation by L in the y direction only up to a gauge transformation:

$$\psi(x, y) \rightarrow \psi(x, y)e^{ixy} \quad (28)$$

Therefore we have the following periodicity conditions on $f(z)$:

$$f(z + L) = f(z) \quad , \quad f(z + iL) = e^{\pi N_s} e^{-i2\pi \frac{N_s}{L} z} f(z) \quad (29)$$

Let us introduce the "magnetic" translation operators \mathcal{S} and \mathcal{T} , defined by:

$$\mathcal{S}_a f(z) = f(z + a) \quad \mathcal{T}_b f(z) = e^{-b^2/2 + ibz} f(z + ib) \quad (30)$$

which satisfy the commutation relations of the Heisenberg group:

$$\mathcal{S}_a \mathcal{T}_b = e^{iab} \mathcal{T}_b \mathcal{S}_a \quad (31)$$

The independent solutions of the periodicity conditions, eq.29, form a basis for the N_s -dimensional representation of the discrete subgroup of magnetic translations generated by \mathcal{S}_{L/N_s} and \mathcal{T}_{L/N_s} and can be taken as the Theta-functions with rational characteristics l/N_s where $l = 1, 2, \dots, N_s$ ¹⁷:

$$f_l(z) = \Theta \left[\begin{matrix} l/N_s \\ 0 \end{matrix} \right] \left(z \frac{N_s}{L} | iN_s \right) \quad (32)$$

This discussion can be easily generalized¹⁸ to the case of an arbitrary magnetic field, provided the flux quantization condition is verified. We will not enter into an

analysis of this point that will become evident from the discussion of the specific case of interest given below.

It is also interesting to establish the connection of the states given by eq. 32 with a set of *generalized coherent states*¹⁹. Define the generators of the generalized coherent states as

$$\mathcal{D}_l(b, b^\dagger | N_s) = \sum_{r \in Z} e^{(\alpha_r^l b^\dagger - \bar{\alpha}_r^l b)} = \sum_{r \in Z} e^{-\frac{|\alpha_r^l|^2}{2}} e^{\alpha_r^l b^\dagger} e^{-\bar{\alpha}_r^l b} \quad (33)$$

where $\alpha_r^l = iL(r + l/N_s)$ and b, b^\dagger are in the FB representation, eq 11. Then

$$f_l(z) = \mathcal{D}_l(b, b^\dagger | N_s) \chi_0 \quad (34)$$

where $b\chi_0 = 0$.

Then, at filling $\nu = N_e/N_s = 1$, the analytic part of the Laughlin wave function on the torus can be written as

$$f_{\nu=1}(z_1, z_2, \dots, z_{N_e}) = \sum_P (-1)^P \mathcal{D}_{i_1}(b_1, b_1^\dagger | N_e) \mathcal{D}_{i_2}(b_2, b_2^\dagger | N_e) \dots \mathcal{D}_{i_{N_e}}(b_{N_e}, b_{N_e}^\dagger | N_e) \chi_0 \quad (35)$$

Here $i_1, i_2 \dots i_{N_e}$ is a permutation, P , of the sequence of natural numbers not bigger than the electron number N_e .

On the basis of the periodicity properties of the Θ 's and of the location of the zeros it can be seen that¹

$$f_{\nu=1}(z_1, z_2, \dots, z_{N_e}) = \Theta \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right] \left(\frac{Z}{L} | i \right) \prod_{i < j} \Theta_1(z_{ij} | i) \quad (36)$$

where $Z = \sum_{i=1}^{N_e} z_i$ is the "center of charge" coordinate, $z_{ij} = (z_i - z_j)/L$ and $\Theta_1 = \Theta \left[\begin{smallmatrix} 1/2 \\ 1/2 \end{smallmatrix} \right]$.

Let us now add to the previous uniform magnetic field an infinitely thin flux tube located at z' of strength $-2k$. Then the single particle wave functions may be written as

$$\psi(x, y) = e^{-\frac{1}{2}y^2 + h(z)} g(z) \quad (37)$$

where, with suitable normalization,

$$h(z) = 2k \ln \frac{\Theta_1(\frac{z-z'}{L} | i)}{\Theta_1(0 | i)} \quad (38)$$

¹For simplicity we limit ourselves to N_e odd. The case N_e even would require only a slight generalization of eq. 36.

Taking into account the transformation properties

$$\Theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (w+1|\tau) = e^{2\pi i \alpha} \Theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (w|\tau) \quad (39)$$

$$\Theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (w+\tau|\tau) = e^{-2\pi i \beta} e^{-\pi i \tau} e^{-2\pi i w} \Theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (w|\tau) \quad (40)$$

one gets the following boundary conditions for g

$$g(z+L) = g(z) \quad (41)$$

$$g(z+iL) = g(z) e^{\pi(N_s-2k)} e^{-i2\pi \frac{N_s}{L} z} e^{i2\pi \frac{2k}{L} (z-z')} \quad (42)$$

which, due to eqs. 39 and 40, are satisfied by

$$g_l(z) = \Theta \left[\begin{smallmatrix} l/(N_s-2k) \\ 0 \end{smallmatrix} \right] \left(z \frac{N_s-2k}{L} + z' \frac{2k}{L} | i(N_s-2k) \right) \quad (43)$$

When we extend the discussion to the system of Jain's composite fermions and study the motion of the i^{th} particle in the presence of all the others we have

$$\psi(x_i, y_i) = e^{-\frac{1}{2}y_i^2 + h(z_i)} g(z_i) \quad (44)$$

where

$$h(z_i) = 2k \sum_{j \neq i} \ln \left[\frac{\Theta_1(z_{ij}|i)}{\Theta'_1(0|i)} \right] \quad (45)$$

with the following boundary conditions

$$g(z_i+L) = g(z_i) \quad (46)$$

$$g(z_i+iL) = g(z_i) e^{\pi N_e} e^{-i2\pi \frac{N_e}{L} z_i} e^{\pi 2k} e^{-i2\pi \frac{2k}{L} Z} \quad (47)$$

It is clear from eqs. 44 and 47 that the wave function of each composite fermion explicitly depend on the position of all the others. To obtain an expression like eq. 35 for the composite fermions, but corresponding to a filling $\nu_{eff} = 1$ as expected in Jain's picture, we introduce a collective vacuum

$$\chi_c(z_1, z_2, \dots, z_{N_e}) = \chi(Z) \prod_{i \neq j} \left[\frac{\Theta_1(z_{ij}|i)}{\Theta'_1(0|i)} \right]^{2k} \quad (48)$$

or, in other terms, we introduce the operators b_i and b_i^\dagger

$$b_i = \partial_i - 2k\partial_i \sum_{j \neq i} \ln \left[\frac{\Theta_1(z_{ij}|i)}{\Theta'_1(0|i)} \right] - \partial_i \ln \chi(Z) \quad , \quad b_i^\dagger = z_i \quad (49)$$

so that $b_i \chi_c = 0$. The dependence on the center of charge variable, that we have left unspecified, must be consistent with eq. 47 and with the requirement that the multi-particle wave functions lead to a uniform charge distribution. These wave functions take now the same form as for ordinary fermions at $\nu = 1$, eq. 35, i.e.

$$f(z_1, z_2 \dots z_{N_e}) = \sum_P (-1)^P \mathcal{D}_{i_1}(b_1, b_1^\dagger | N_e) \mathcal{D}_{i_2}(b_2, b_2^\dagger | N_e) \dots \mathcal{D}_{i_{N_e}}(b_{N_e}, b_{N_e}^\dagger | N_e) \chi_c \quad (50)$$

where the D 's are still given by eq. 33 with b_i and b_i^\dagger given by eq. 49 and χ_c by eq. 48. Taking into account eq. 36 we have

$$f(z_1, z_2 \dots z_{N_e}) = \prod_{i < j=1}^{N_e} \left[\frac{\Theta_1(z_{ij}|i)}{\Theta'_1(0|i)} \right]^m F(Z) \quad (51)$$

where

$$F(Z) = \chi(Z) \Theta \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right] \left(\frac{Z}{L} | i \right) \quad (52)$$

The analytic part of the multi-particle wave functions, eq. 51, will give the prescribed periodicity condition, eq. 29, in each variable provided that:

$$F(Z + L) = F(Z) \quad , \quad F(Z + iL) = F(Z) e^{\pi m} e^{-i2\pi \frac{m}{L} Z} \quad (53)$$

On the other hand these conditions on $F(Z)$ imply a uniform charge distribution. Indeed one easily sees that eqs. 53 require that the charge density is invariant, both in the x and y directions, under a translation of $\Delta = L/N_e$, which is vanishing small in the thermodynamical limit.

Then the $\nu_{eff} = 1$ wave function for Jain's composite fermions leads to the usual multi-electron wave functions on the torus at filling $\nu = 1/m$ ¹⁰, as eqs.53 are satisfied by the m independent functions:

$$F_l(Z) = \Theta \left[\begin{smallmatrix} l/m \\ 0 \end{smallmatrix} \right] \left(\frac{Zm}{L} | im \right) \quad (54)$$

The novel feature of the degeneracy arises from the different possible choices of the dependence on the center of charge variable consistent with the requirement of an uniform charge distribution.

3. We close this paper with a brief discussion of the FQHE at arbitrary rational filling, limiting ourselves to the case of the plane. Our simple analysis of the electron motion in the presence of a uniform magnetic background and localized flux tubes

can be used to discuss the FQHE at fillings of the form $\nu = p/(2kp + 1)$ as IQHE for Jain's composite fermions at integer filling p . Indeed the appropriate operators a, a^\dagger and b, b^\dagger have commutation relations that differ from those relative to a purely uniform background only by terms that have delta functions in the relative coordinates. As they will be acting on the appropriate collective vacuum states, such as 21, the Landau level structure will be preserved and the wave function for composite fermions at filling p will differ by the ordinary one only by a factor that has a zero of order $2k$ in each relative coordinate. As shown by Jain such a state lies predominantly in the lowest Landau level for N_e large ².

On the other hand this approach can be extended in a way that mimics the traditional hierarchical scheme, by introducing a number of charged excitations N_I at the I^{th} hierarchical level, each carrying its own statistical flux tube. One is then led to introduce a matrix K_{IJ} , describing the couplings of a particle at I^{th} level with the statistical field relative to the J^{th} level or, in other terms, the braiding factors between the corresponding "particles". The discussion of the motion of the i^{th} "particle" of the I^{th} level in the presence of an external magnetic field and of flux tubes located at the positions of all the other "particles" leads naturally to the introduction of the operators:

$$(b_i^I)^\dagger = z_i^I \quad , \quad b_i^I = \partial_i^I - \partial_i^I \sum' H_{IJ} \ln(z_i^I - z_j^J) = \partial_i^I - \sum_{(j,J) \neq (i,I)} \frac{H_{IJ}}{z_i^I - z_j^J} \quad (55)$$

where $H_{IJ} = K_{IJ} - \mathbf{1}$, i and j label the "particles" in a given level while I and J run on the different levels of the hierarchy and \sum' means the sum on all possible couples.

Then one can introduce a "collective vacuum" χ_H defined by $b_i^I \chi_H = 0$ and write a "wave function" along the same line of eq. 13 and 20, namely

$$\chi(\{z_i^I\}) = \prod_I \sum_{P_I} (-1)^{P_I} (b_1^{I\dagger})^{n_{i_1}^I} (b_2^{I\dagger})^{n_{i_2}^I} \dots (b_N^{I\dagger})^{n_{i_{N^I}}^I} \chi_H \quad (56)$$

where $n_{i_1}^I, n_{i_2}^I \dots n_{i_{N^I}}^I$ is a permutation, P^I , of the sequence of non negative integers smaller than the number N^I of "particles" at level I .

It can be seen that this simple approach is in agreement with the general description of the hierarchical model given by means of an abelian Chern-Simons field theory ²⁰ or by using 2D Coulomb gas Vertex operators ²¹. Furthermore, it allows to verify that all the hierarchical states can be classified according to the universality class of a $W_{1+\infty}$ algebra ²², generalizing the definition of the generators, eq. 23, ¹⁶:

$$\mathcal{L}_{m,n} = \sum_{i,I} (b_i^{I\dagger})^{m+1} (b_i^I)^{n+1} \quad , \quad n, m \geq -1 \quad (57)$$

All these different approaches, describing the FQHE in terms of an appropriate set of quasiparticles and relative statistical fields, whose properties are encoded in the matrix of their braiding factors, are essentially kinematical in nature, as they

give all possible quantum Hall fluids. From this point of view the reason why the sequence of fillings selected by Jain's approach appear experimentally as the most prominent one is left unanswered and should be addressed at a dynamical level.

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